UNSTEADY HEAT TRANSFER BY RADIATION IN A SYSTEM OF COAXIAL CYLINDERS

L. P. Batov, Ya. I. Karker, and M. G. Kogan

Inzhenerno-Fizicheskii Zhurnal, Vol. 11, No. 3, pp. 289–295, 1966

UDC 536.33

An investigation has been made on a non-linear analog computer of unsteady heat transfer by radiation in a system of coaxial cylinders simulating the heating of the load component, the muffle, and the lining of a vacuum induction furnace.

An investigation of unsteady heat transfer by radiation in a system of coaxial cylinders has been conducted in connection with development of a series of powerful, low-inertia vacuum heating furnaces. The purpose of the furnaces is oxygen-less, rapid heating of thin-walled metal components during heat treatment, with thorough and flexible control of conditions according to a program, in a wide range of temperature (up to 1500° K and above).

Direct induction heating of components was not used because of their asymmetrical shape and the very exacting demands as regards uniformity of temperature distribution. It therefore proved necessary to introduce a "buffer" element—a cylindrical muffle of heat-stable alloy (Fig. 1), located in the axially symmetric electromagnetic field of an inductor. Both the electromagnetic equipment and the items to be controlled in this kind of furnace differ appreciably from the ordinary heating furnaces in having low inertia, due to the fact that the heat capacity of the component, the muffle, and the heat insulation are about equal. The basic problem in the theory of these furnaces is the investigation of the kinetics of heating with various combinations of the physical and geometrical combinations of all the items participating in the heat transfer. It is also important to determine the optimum heat balance, since the power of the furnace is in the hundreds of kilowatts.

The investigation of heat transfer by radiation between several bodies over a wide temperature range is a substantially non-linear problem, and there are no effective analytical methods of solving it. Therefore the final calculations for the furnace were made by approximate numerical methods using a computer, while a preliminary analysis of the heating conditions and synthesis of the control schemes was done according to the results of modeling. Since it is very difficult to retain the physical nature of the process under examination while modeling with dummies, and comparison with actual furnaces of different power levels permitted only crude estimates to be made, our work was based mainly on modeling, based on the analogy between thermal and electrical phenomena.

In constructing a model the following simplifications were made:

1. A component of complex shape was replaced by an equivalent cylinder, to which was attributed the physical parameters and the mean geometrical dimensions of the actual item.

2. It was assumed that all axial dimensions were considerably larger than the radial dimensions, which allowed us to neglect edge effects, and to consider that the radiative heat flux was directed only along the radii.

3. The component and the muffle are thin-walled, which allows us to neglect the temperature drop through them.

4. The component and the muffle are made of an alloy for which we need not take into account temperature dependence of thermophysical properties, and can assume constant values for the whole temperature range used.

5. The lining was divided into several layers within each of which the temperature is regarded as uniformly distributed; for this reason the differential equation of heat conduction in partial derivatives was reduced to a system of ordinary differential equations (the method of straight lines). The values of the thermophysical parameters of the lining were determined by three successive approximations. In the first approximation, room temperature values of the parameters were assumed for all the layers. In the next two approximations, the values assumed for each layer were those corresponding to the mean temperature in the layer, as determined by modeling the heating process in the previous approximation.

Thus, the substitution scheme for the furnace consists of n + 2 coaxial cylinders: the component 1, the muffle 2, and n layers of lining. Heat transfer by radiation according to the Stefan-Boltzmann law for gray bodies occurs between the component, the muffle, and the inner layer of the lining, while heat transfer by conduction occcurs between the layers of the lining. Joule heating is generated only in the muffle (which practically completely screens the component from the action of the electromagnetic field).

The energy transfer is described by a system of nonlinear differential equations

$$C_i \frac{\partial T_i}{\partial \tau} = q_{\text{em}.i} + q_{i-1,i} + q_{i+1,i}, \quad i = 1, 2, \ldots, n+1, \quad (1)$$

$$q_{\text{em}.i} = \begin{cases} \pi D_i l p_{\text{em}}, & i = 2 \\ 0, & i \neq 2 \end{cases}, \quad (2)$$

$$q_{i-1,i} = -q_{i,i-1} =$$

$$= \begin{cases} \gamma_{i-1,i}(T_{i-1}^4 - T_i^4), & i = 1, 2, 3 \\ \beta_{i-1,i}(T_{i-1} - T_i), & i = 4, 5, \ldots, n+2 \end{cases}, \quad (3)$$

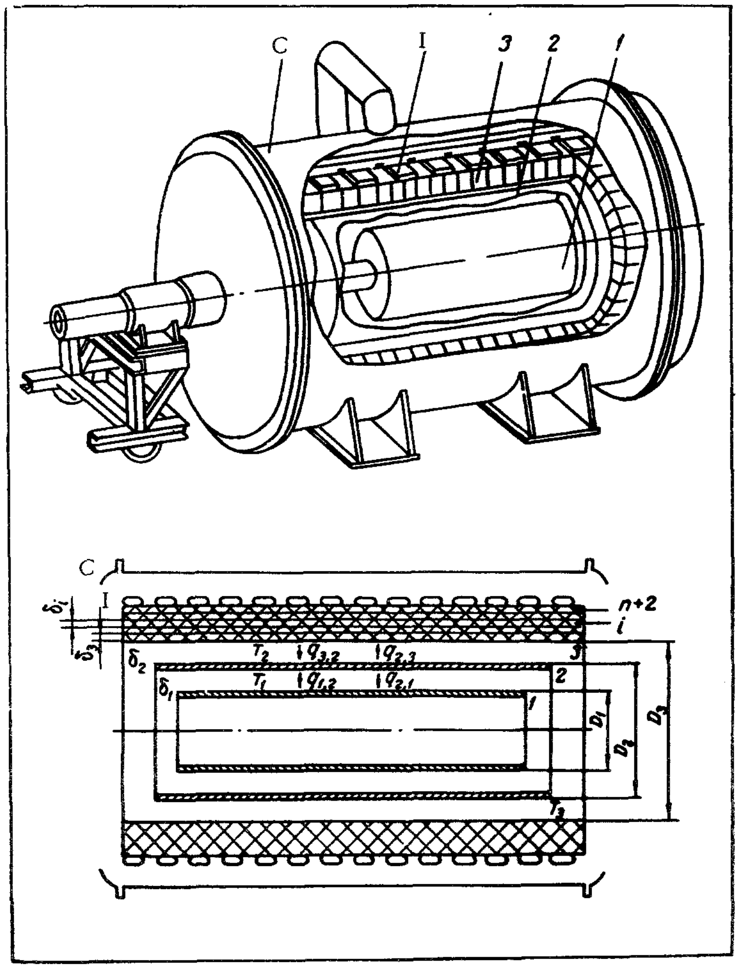

Fig. 1. The induction vacuum heating furnace and a cross sectional view: 1) the component to be heated; 2) the muffle; 3) layer of lining; I) inductor; C) case.

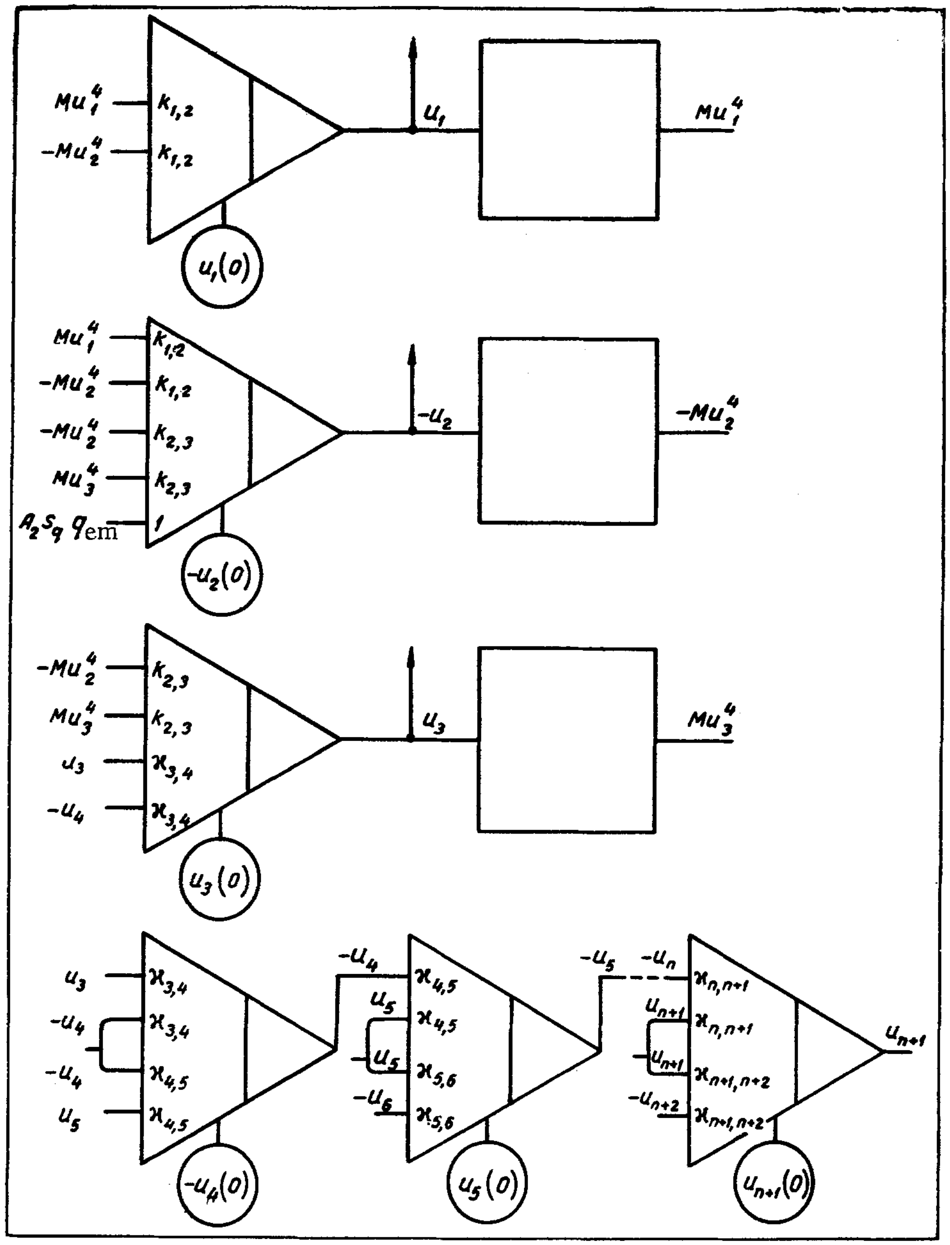

Fig. 2. Solution scheme of the modeling algorithm. The outputs to the recording instruments are shown by arrows. The triangles denote integrating elements, and the rectangles denote non-linear elements.

$$C_i = \pi D_i l\, \delta_i \rho_i c_i, \tag{4}$$

$$\gamma_{i-1,\, i} = \gamma_{i,\, i-1} = \pi D_{i-1} l\, \sigma \times$$

$$\times \left[\frac{1}{\varepsilon_{i-1}} + \left(\frac{1}{\varepsilon_i} - 1 \right) \frac{D_{i-1}}{D_i} \right]^{-1} \tag{5}$$

$$\beta_{i-1,\, i} = \beta_{i,\, i-1} = \pi D_{i-1} l\, \lambda_i / \delta_i. \tag{6}$$

The boundary conditions are: the net flux of thermal radiation to the inside surface of the component (which forms a closed shell) is zero because of symmetry, i. e., $q_{0,1} = 0$; the outer layer of the lining has the temperature of the inductor, which is water-cooled, i. e., $T_{n+2} = T_{ind} = \text{const}$.

The initial conditions are: at the first heating the temperatures of all the items are the same; upon repeated heating the initial temperature distribution is nonuniform (it is determined from the results of modeling of the heating and cooling process in the previous cycle).

In the analog computer the dependent variables are the temperatures T of the items and the heat fluxes q—represented by electrical potentials u and v, while the independent variable is τ—represented by computer time ϑ. The variables determining the processes in the model and in its analog are related by the scale factors

$$u_i = S_T T_i, \quad v_{i-1,\, i} = S_q q_{i-1,\, i}, \quad \vartheta = S_\tau \tau. \tag{7}$$

Following introduction of the computer variables, the system (1)–(3) takes the form

$$\frac{\partial u_i}{\partial \vartheta} = A_i (v_{\text{em},i} + v_{i-1,\, i} + v_{i+1,\, i}),\ i = 1, 2, \ldots,\ n+1, \tag{1a}$$

$$v_{\text{em},\, i} = \begin{cases} S_q q_{\text{em},\, 2}, & i = 2 \\ 0, & i \neq 2 \end{cases}, \tag{2a}$$

$$v_{i-1,\, i} = -v_{i,\, i-1} =$$

$$= \begin{cases} \Gamma_{i-1,\, i} (u_{i-1}^4 - u_i^4), & i = 1, 2, 3 \\ B_{i-1,\, i} (u_{i-1} - u_i), & i = 4, 5, \ldots, n+2 \end{cases}, \tag{3a}$$

where

$$A_i = \frac{1}{C_i} \frac{S_T}{S_q S_\tau}, \quad B_{i-1,\, i} = \beta_{i-1,\, i} \frac{S_q}{S_T}, \quad \Gamma_{i-1,\, i} = \gamma_{i-1,\, i} \frac{S_q}{S_T^4},$$

$$A_i B_{i-1,\, i} = \frac{\beta_{i-1,\, i}}{C_i S_\tau} = \varkappa_{i-1,\, i} = \varkappa_{i,\, i-1}, \quad A_i \Gamma_{i-1,\, i} = k_{i-1,\, i} M,$$

$$M = \frac{1}{S_\tau S_T^3}, \quad \frac{\gamma_{i-1,\, i}}{C_i} = k_{i-1,\, i} = k_{i,\, i-1}.$$

The scheme of solution of this system is shown in Fig. 2. For each of the first three items—the component, the muffle, and the inside layer of the lining—the circuit consists of two units: an integrating and a non-linear unit (in which the variable u_i is raised to the fourth power), while for the succeeding layers of the lining there are only integrating elements. In the event that a combined thermal insulation system is used, consisting of screens as well as the lining (one of the modifications of the furnaces), additional circuits are included in the solution scheme, with integrating and non-linear units (according to the number of screens). The group of equations and the solution scheme form a modeling algorithm, which we regard as a general solution of the class of problem being examined. Particular solutions are obtained after inserting the coefficients k and $\varkappa$ into the computer, corresponding to specific values of the physical and geometrical parameters of the items.

As an example, Fig. 3 shows typical curves, obtained during modeling of a heating process in a 500 kW furnace. The component was a metal cylinder of wall thickness $\delta_1 = 3 \cdot 10^{-3}$ m and diameter $D_1 = 1.15$ m; density $\rho_1 = 7.8 \cdot 10^3$ kg/m^3; specific heat $c_1 = 635$ J/kg · degree. The muffle was a cylinder with wall thickness $\delta_2 = 1.5 \cdot 10^{-3}$ m and diameter $D_2 = 1.25$ m; density $\rho_2 = 7.8 \cdot 10^3$ kg/m^3; specific heat $c_2 = 635$ J/kg · degree. The lining had internal diameter $D_3 = 1.45$ m and was divided into 5 layers, each of thickness $\delta_i = 13 \cdot 10^{-3}$ m; its specific heat was $c_3 = (814 + 230 \cdot 10^{-3} T_m)$ J/kg · degree; its thermal conductivity was $\lambda = (0.22 + 2.55 \cdot 10^{-4} T_m)$ W/m. degree; its density was $\rho_3 = 1000$ kg/m^3.

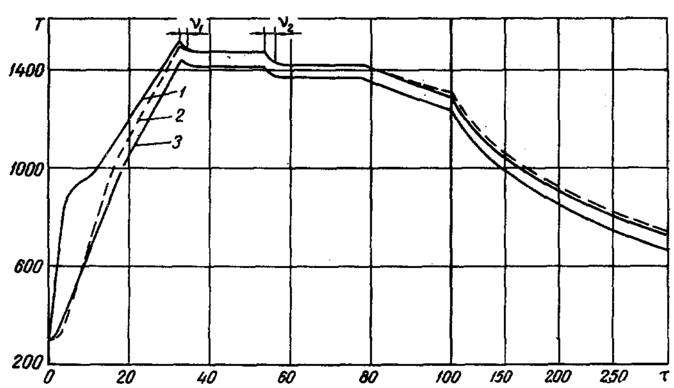

Fig. 3. Variation of the temperatures of 1) the muffle, 2) the component, and 3) the inner surface of the lining. T in °K; τ in min; $\nu_1 = 2.0$ min; $\nu_2 = 2.3$.

After investigation of a number of variants, it was established that the optimum is as follows: begin heating at constant specific power $q_{em} = 3.2 \cdot 10^4$ W/m^2 until the muffle reaches the assigned temperature of T = 1520° K, and then hold it at this temperature with automatic control of the power generated. A characteristic feature of the first stage is a sharp decrease of the rate of heating of the muffle at temperatures when the net radiation density (from both surfaces) approaches the specific power of the induced current. Following some slowing down, the rate of heating of the muffle again increases, due to the fact that the temperatures of the component and of the lining gradually increase, and their effective radiation begins to have an appreciable influence on the heat transfer balance. In the final stage the temperatures of the component and of the muffle encircling it become close together, and the heating proceeds as if they were one item with the heat capacity of their sum.

At the start of heating the power is approximately twice as great as at the end. With increase of this ratio (of initial to final power) the installed power of the furnace increases, but the duration of the process is essentially not shortened, since even when the muffle momentarily attains its working temperature, the heating rate of the component cannot exceed the limit imposed by the thermophysical and geometrical parameters. Decrease of the initial power leads to lengthening of the heating process, which is unfavorable on technical grounds.

Cooling of the component and of the furnace takes place in vacuum down to a temperature at which oxidation of the metal can no longer occur when air is admitted into the chamber. The radiation heat transfer is evidently described in this case also by the system (1)–(3), under the condition that $q_{em} = 0$. The duration of the cooling in a typical case may be gauged from Fig. 3.

Modeling on analog computers is a very effective means of investigating the influence of individual elements on the dynamic characteristics of electro-thermal equipment and of solving problems concerning the make-up of furnaces and of automatic control systems. The transfer and frequency characteristics may be obtained and analyzed directly after setting up the next variant of the parameters of the solution scheme (and when the solutions are made periodic [1]—during the modeling process). In particular, the time constant of the charged furnace is easily evaluated from the response of the output signal to a step in voltage—the analog of the generated power. Due to nonlinearity of the transfer equations, the time constant ν depends appreciably on the control level, increasing with reduction of temperature (Fig. 3). Further investigation of the dynamic characteristics of the furnace as a multi-capacitance control item indicates that the beginning of the transition process is determined by the parameters of the muffle, while its final stage is determined by the totality of the parameters of all elements of the system.

NOTATION

T is the absolute temperature; τ is the time; ν is time constant; $q_{i-1,i}$ is the density of net heat flux from item i − 1 to item i; q_{em} is the specific power of the induced currents (referred to unit area); C is the heat capacity; c is the specific heat; λ is the thermal conductivity; ρ is the density; ε is the emissivity; σ is the Stefan-Boltzmann constant; l is the length; D is the diameter; δ is the wall thickness; u and v are electrical voltages—the analogs of temperature and heat flux density; ϑ is the computer time; S is the scale factor.

REFERENCE

1. R. Tomovich and U. Karplyus, High-Speed Analog Computers [in Russian], Izd. Mir, 1964.

28 March 1966